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Economics of Exchange Rates

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Number of questions: This exam consists of 2 questions.

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1. **Portfolio Balance Model**

   This question covers the portfolio balance model and is related to the learning objective: describe and use the portfolio balance and the signaling models to analyze the effects of policy interventions (central bank interventions, monetary and fiscal policy) on the exchange rate.

   Consider the standard Portfolio Balance Model comprised of the following functions

$$W \equiv M + B_p + SF_p \tag{1}$$

$$M = m\left(r, E\dot{s}, Y, W\right) \quad m_r < 0, m_{E\dot{s}} < 0, m_y > 0, m_w > 0 \tag{2}$$

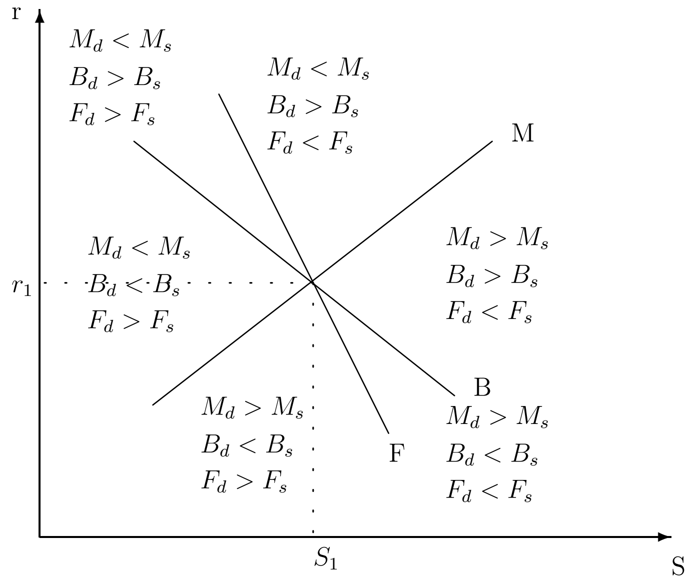
$$B_p = b\left(r, E\dot{s}, Y, W\right) \quad b_r > 0, b_{E\dot{s}} < 0, b_y < 0, b_w > 0 \tag{3}$$

$$SF_p = f\left(r, E\dot{s}, Y, W\right) \quad f_r < 0, f_{E\dot{s}} > 0, f_y < 0, f_w > 0 \tag{4}$$

   Notation is standard.

   (a) Consider the short-term version of this model, i.e., we assume that prices are constant. Illustrate the model above in a graph and provide intuitive explanations of the slopes of the three short-run asset market equilibrium curves.

   Solution: As is standard, we illustrate the model in the interest rate–exchange rate plane, see the graph below. The portfolio balance model is in equilibrium when all three markets are in equilibrium, i.e., where the three schedules intersect.



The M–schedule is upward sloping and describes equilibrium in the domestic money market. The explanation is that a depreciation of the exchange rate (an increase in S) leads to an increase in the domestic investor's wealth (foreign assets are worth more after the depreciation). The increase in wealth leads to an increase in the demand for money. But since the money supply is fixed, the increase in the money demand can only be offset by an increase in the interest rate.

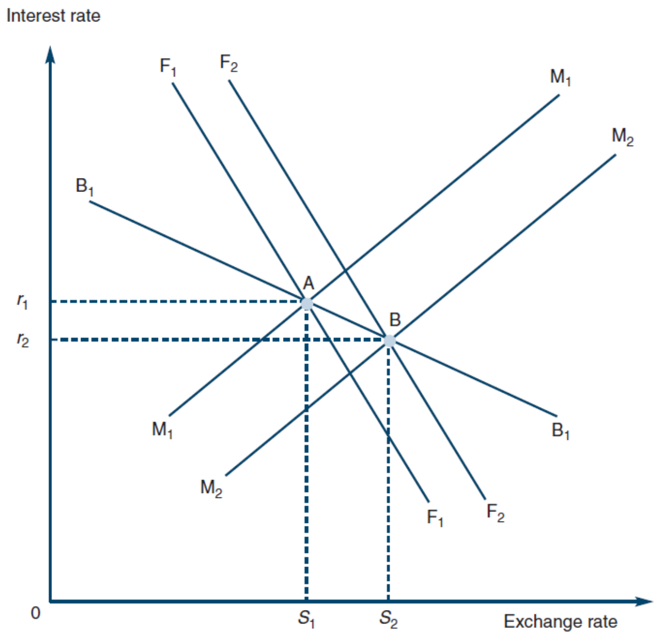
The B–schedule is downward sloping since a depreciation that raises wealth also raises the demand for domestic bonds increases the price of bonds leading to a fall in the interest rate which will reduce the demand for domestic bonds. A depreciation must then be offset by a fall in the interest rate.

The F–schedule depicting equilibrium on the market for foreign bonds is also downward sloping. The reason for this is that a depreciation of the exchange rate leads to an increased demand for domestic bonds and therefore investors are inclined to sell money and foreign bonds to buy domestic bonds. Alternatively, a rise in the interest rate makes domestic bonds more attractive and the exchange rate must depreciate in order to maintain equilibrium on the market for foreign bonds, i.e., to increase the value of foreign bonds measured in the domestic currency.

(b) Use the graphical illustration of the model to analyze the effects of a non-sterilized foreign exchange operation where the central bank operates to depreciate the home currency. First, explain the central bank strategy and then use the graphical illustration to find how the interest rate and the exchange rate will be affected.

Solution: In order to depreciate the currency, the central bank buys foreign bonds from the private sector and sells money. This implies that $\mathrm{d}M = -S\mathrm{d}F_p = S\mathrm{d}R$. Excess supply of money leading to a shift in the M schedule down to the right to M_2. Excess demand for foreign bonds: The F schedule shifts up to the right. The shortage of foreign bonds in the portfolios requires the exchange rate to depreciate. The domestic currency value of investor's remaining holdings of foreign bonds increase. This is illustrated in the graph below.

We find that the total effect of a sterilized foreign exchange operation is a depreciated currency (the target of our operation) and a lower interest rate.



(c) We can solve the portfolio balance model given the assumptions that $dY = dE\dot{s} = 0$ to find that

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \frac{1}{D} \begin{bmatrix} F_p m_w & 0 & -b_w F_p \\ -m_r F_p & 0 & b_r F_p \\ f_r m_w - f_w m_r & -b_r m_w + b_w m_r & b_r f_w - b_w f_r \end{bmatrix} \begin{bmatrix} dB_p \\ SdF_p \\ dM \end{bmatrix}$$

where $D = b_r m_w F_p - m_r b_w F_p$. Use the model above to derive the analytical effects of a non-sterilized intervention aimed at depreciating the currency. Explain carefully.

Solution: As stated above, a non-sterilized foreign exchange operation implies that $dB_p = 0$, $SdF_p = -dM$. Insert this into our analytical solution above such that

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \frac{1}{D} \begin{bmatrix} F_p m_w & 0 & -b_w F_p \\ -m_r F_p & 0 & b_r F_p \\ f_r m_w - f_w m_r & -b_r m_w + b_w m_r & b_r f_w - b_w f_r \end{bmatrix} \begin{bmatrix} 0 \\ -dM \\ dM \end{bmatrix}$$

This implies that

$$dr = -\frac{b_w}{b_r m_w - b_w m_r} < 0$$

since the $b_r m_w - b_w m_r$ is positive and b_w is positive.

$$dS = (b_r(f_w + m_w) - b_w(f_r + m_r)) \frac{dM}{F_p} > 0$$

since b_r and b_w are both positive, $D > 0$ and f_r and m_r are both negative. Finally,

$$dW = b_r F_p \frac{dM}{D} > 0$$

since $b_r > 0$ and $D > 0$.

(d) The central bank fears that inflation will increase as a result of the depreciated currency. Therefore they consider to use a sterilized intervention instead. Use the model above to derive the analytical effects of a non-sterilized intervention aimed at depreciating the currency. Is the effect on the exchange rate larger if domestic and foreign bonds are poor substitutes? Explain carefully.

Solution: In this case we have that $dB = dB$, $SdF_p = -dB$ and $dM = 0$. Our solution then implies

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \frac{1}{D} \begin{bmatrix} F_p m_w & 0 & -b_w F_p \\ -m_r F_p & 0 & b_r F_p \\ f_r m_w - f_w m_r & -b_r m_w + b_w m_r & b_r f_w - b_w f_r \end{bmatrix} \begin{bmatrix} dB \\ -dB \\ 0 \end{bmatrix}$$

implying that

$$dr = m_w F_p \frac{dB}{D} > 0$$

since $m_w > 0$. The effect on the exchange rate is

$$dS = (f_r m_w - f_w m_r + b_r m_w - b_w m_r) \frac{dB}{D}$$

From the wealth identity in equation (1) we find that the total differential with respect to W implies that

$$0 = (1 - m_w - b_w - f_w) \, dW$$

and that the total differential with respect to r holding W constant is given by

$$0 = (m_r - b_r - f_r) \, dr$$

Use these relations to rewrite the effect of the sterilized foreign exchange operation on the exchange rate as

$$dS = -m_r \frac{dB}{D} > 0$$

since $m_r < 0$. If domestic and foreign bonds are poor substitutes, then we expect the money demand to be highly interest-sensitive and the effect on the exchange rate should increase. The efficiency of the sterilized foreign exchange operation increases.

2. **Microstructure model**

This question relates to the learning objectives: describe and use microstructure based models (rational expectations and portfolio shift models) to analyze price determination on the foreign exchange market and summarize the empirical evidence on these models;

describe and use the portfolio shift model to analyze the effects of news (macro data releases and central bank interventions) on the exchange rate.

The model used in the question combines microstructure and macro perspectives to allow for an analysis of the effects of how order spot exchange rates are determined and how order flow may affect the quotes. The starting point of the analysis is a standard macro model of the spot exchange rate which derived from the micro-based model discussed in Evans.

(a) Give a short overview of how the foreign exchange market is organized and explain how order flows affect quotes.

Solution:

- The FX market is decentralized and fragmented and can be divided into two types of markets, the retail market (customer–to–bank or broker market) and the interbank market (which can be divided into two parts; the direct market (bank–to–bank) and the indirect or brokered market (bank–to–broker)).
- The direct market is a double–auction (both buy and sell prices are specified) and open–bid market (all offers to buy or sell at a specified price are announced to all market participants) whereas the indirect or brokered market is a single–auction (either buy or sell prices are quoted), closed–order–book or limit–book (offers only known to agents placing the order of either buying or selling a certain amount of currency against another currency at a specified rate of exchange).
- Dealers face two forms of the constraints: information and position constraints (no net overnight position). The former describe the information available to dealers as they make their trading decisions, the latter are the instruments used by banks to limit the risk of trading losses.
- Dealers face uncertainty about overall state of the market. They cannot observe the complete structure of limit orders on the electronic brokerages and they do not know what is being traded directly. Dealers at some large banks have access to information on the customer orders received by their customer trading desks world-wide. At best, this gives them a partial picture of the aggregate currency orders hitting the market.
- Customer orders fall into two categories (i) real trade related orders and finance related trades.
- Order flow is defined as: the difference between purchase and sell orders initiated by customers during a trading period. Order flows can be both positive and negative. For example: The order flow when a customer is selling 10 units of a currency is -10 (the transaction volume is 10). The order flow if a customer is buying 10 units of a currency is +10 (the transaction volume is 10). A positive order flow indicates to the dealer that customers value foreign currency more than the dealers asking price. A negative order flow indicates that customers value foreign currency less than the dealers bid price.

- If there is a macroeconomic announcement (which is publicly observed), the customer values this new information and based on this valuation places either a buy or a sell order to the bank. This gives information about the new announcement that will affect prices on the interbank market, the order flow shows how the customer values new information. If all information about fundamentals were publicly known, then order flows cannot signal information about fundamentals.

(b) Consider the following version of the micro-based exchange rate model. Assume that all dealers quote the same spot price to both other dealers and to their customers (quotes are publicly announced) and assume that the exchange rate is determined by fundamentals (as in most macro based models)

$$s_t = (1-b)\,\Sigma_{i=0}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]$$

Give a short motivation to this equation and explain how the spot exchange rate is determined.

Solution: s_t is the log price of currency quoted by all dealers, $0 < b < 1$ is a discount factor, f_{t+i} is the exchange rate fundamental at time $t+i$ and Ω_t^D is the information common to all dealers at the start of period t. This model can be derived using any standard macro model of the exchange rate, the only difference between different types of monetary models is what constitutes the fundamentals, i.e., what fundamental macro variables should be included in f. In the micro-based model in Evans (2010) paper, the risk premium is also included explicitly. Otherwise the model in the question is fully consistent with the micro-based model.

(c) Show that the model above can be re-written as

$$\Delta s_{t+1} = \frac{1-b}{b}\left(s_t - \mathrm{E}\left[f_t \mid \Omega_t^D\right]\right) + \varepsilon_{t+1}$$

by decomposing a change in the exchange rate into the unexpected and expected change. How is the spot exchange rate determined? Explain carefully how order flows affect the spot rate. Do public and private information available to customers affect spot rates?

Solution: First, we note that the change in the spot exchange rate from t to $t+1$ can be expressed as

$$s_{t+1} - s_t = \Delta s_{t+1} = \overbrace{s_{t+1} - \mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right]}^{\text{unexpected change}} + \overbrace{\mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t}^{\text{expected change}}$$

Start with the unexpected change. From the model given in the question we know that

$$s_{t+1} = (1-b)\sum_{i=0}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] = \frac{1-b}{b}\sum_{i=1}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right]$$

Take the expected value of this expression given information available at time t

$$\mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right] = \frac{1-b}{b} \sum_{i=1}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]$$

and take the difference such that the unexpected change is given by

$$s_{t+1} - \mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right] = \frac{1-b}{b} \sum_{i=1}^{\infty} b^i \left(\mathrm{E}\left[f_{t+i} \mid \Omega_{t+i}^D\right] - \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)$$

Next, we compute the expected change in the spot rate from t to $t+1$ given information available at time t, i.e.,

$$\mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t = \frac{1-b}{b} \sum_{i=1}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right] - (1-b) \sum_{i=0}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]$$

Simplify this expression and add and subtract $(1-b)\,\mathrm{E}\left[f_t \mid \Omega_t^D\right]$ to find that the expected change can be written as

$$\mathrm{E}\left[s_{t+1} \mid \Omega_t^D\right] - s_t = \frac{1-b}{b} \left(\overbrace{(1-b) \sum_{i=0}^{\infty} b^i \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]}^{s_t} - \mathrm{E}\left[f_t \mid \Omega_t^D\right] \right)$$

Take the sum of the unexpected and expected changes to arrive at the result

$$\Delta s_{t+1} = \frac{1-b}{b} \left(s_t - \mathrm{E}\left[f_t \mid \Omega_t^D\right]\right) + \varepsilon_{t+1}$$

where

$$\varepsilon_{t+1} = \frac{1-b}{b} \sum_{i=1}^{\infty} b^i \overbrace{\left(\mathrm{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]\right)}^{\text{new information}}$$

We have now decomposed Δs_{t+1} into two parts, the expected change and the unexpected change.

- Our model above suggest that spot exchange rates are determined by current and future fundamentals. Therefore, spot rates must include forecasts of future fundamentals given common knowledge Ω_t^D.
- Our model suggest that order flows contain information about future fundamentals that is not public, i.e., not in Ω_t^D. Then order flows should predict future fundamentals beyond information contained in Ω_t^D! In other words, order flows should add to the forecasting power of all variables in Ω_t^D.
- Both components are important! Dealers period–t quote must be based on public information known at time t, i.e., $\mathrm{E}\left[\Delta s_{t+1} \mid \Omega_t^D\right]$. Unexpected changes reflect new information arriving between the start of period t and $t+1$. But new information is only important if it revises dealers forecast of the present value of fundamentals based on common information, i.e., $\mathrm{E}\left[f_{t+i} \mid \Omega_{t+1}^D\right] - \mathrm{E}\left[f_{t+i} \mid \Omega_t^D\right]$.

- The curriculum highlights that market participants are not all alike. Dealers may not respond in the same way to new information about fundamentals. Common knowledge (included in Ω_t^D) which is simultaneously observed by all dealers will immediately be incorporated into quoted prices. From the empirical literature covered during the course we know that it usually takes about a minute before new information is reflected in the price. However, in some cases the effects may be long-lasting, i.e., last up to a few days after the news announcement. This is consistent with the fact that market participants are not all alike, it may be that dealers interpret common knowledge differently. Two dealers may not use the same model linking fundamentals to spot rates. This implies that new information which is common knowledge can be the source of dispersed information, i.e., be a source of customer order flows. Common knowledge (macro announcements) may operate both via the direct channel (common knowledge to all dealers) and via the indirect channel (dispersed information) through order flows.
- Order flows have no immediate effect on quotes because it is private information. But, if the order flow is public knowledge to all dealers, there will be an effect.Private information is transmitted between dealers through interdealer orders. Dealers use private information to trade in the interbank market creating order flows between dealers which will transmit and make private information public (information aggregation).

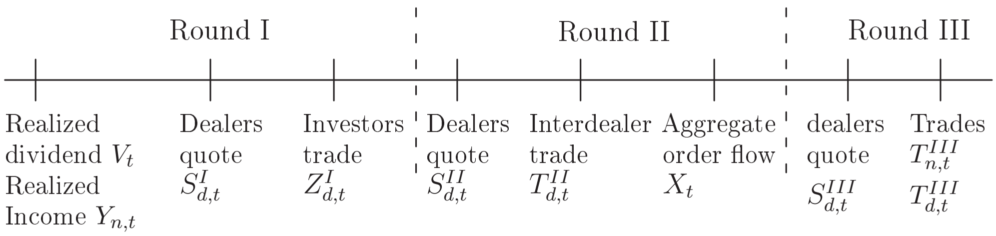
(d) Explain briefly the underlying assumptions of the portfolio shift model.

Solution: The basic underlying assumptions are:

- Customer orders on the retail market provides private information to the dealers. Dealers working at large banks with a large customer base have informational advantages.
- Customer orders are generated by different types of agents and for different reasons, for example for speculation or risk–management.
- The broker plays an important role in the model since it absorbs the imbalance of trades among dealers in such a way that dealers can achieve their desired holdings of foreign exchange. The broker also allows each dealer to hold no foreign exchange inventory overnight.
- There are two assets in the model, one risky asset (the foreign exchange) and one risk–free asset with a daily return equal to $1 + r$.
- The portfolio shift model describes how trades on the retail and the interbank markets relates to the spot exchange rate.
- There are D risk–averse dealers indexed by d, there is a continuum of risk–averse investors (customers) indexed by $n \in [0, 1]$.
- All trading decisions are motivated by the desire of each agent to maximize expected utility, neither dealers or investors are motivated to trade for information.

(e) Use the portfolio shift model to explain how spot exchange rates are determined and pay special attention to how order flows affect quotes. Compare the predictions of the portfolio shift model with the macro based model in question (b).

Solution: The basic portfolio shift model is illustrated below. The time line should be thought of as one trading day.



- We divide one trading day into three rounds, in Round I dealers quote prices and receive orders from customers (the retail market), in Round II dealers trade among themselves and with the broker to share inventory risk and in Round III the retail market reopens and all three market participants trade simultaneously in order to share inventory risk more broadly.
- The market opens and all customers and dealers observe the current payoff on foreign exchange V_t which represents the arrival of public news and customers receive income denominated in foreign currency $Y_{n,t}$ which is private information. The private information generates hedging motives for customer orders. Dealers enter the market with holdings of foreign currency and wealth (the sum of domestic and foreign currency holdings).
- Each dealer independently and simultaneously quotes a scalar price to his/her customers, $S^I_{d,t}$ at which the dealer will buy or sell currency. (Adding bid–ask spreads would not affect the main arguments.) All dealers (and the broker) must quote the same price since a dealer deciding to quote a different price would be exposed to arbitrage trading losses. At any point in time, there is only one equilibrium spot exchange rate.
- The price is publicly observed by all dealers and investors and are good for orders of any size. The quotes are determined before the dealers observe quotes by other dealers and are based on information available from the previous day and the new public information transmitted through the current payoff from foreign exchange holdings.
- Investors place their orders and these could be placed with more than one dealer. Customer orders are only observed by the dealer implying that total demand is unobserved. The customer orders received by a dealer is denoted $Z^I_{d,t}$ and is positive for net customer purchases and negative for net customer sales.

- It is important to distinguish between the two types of information in the model, the public information (dividends) and the private information (income).The public information arrives at the start of each trading day and are immediately incorporated into the quotes in round I. The information embedded into the private information cannot affect the quotes. Investors place their orders based on their own income. Each dealer receives a fraction of the aggregate customer orders and therefore receives a noisy signal about the aggregate component of customer income.
- Each dealer and the broker simultaneously and independently quotes a price to other dealers, $S_{d,t}^{II}$. Dealers act on the information they received through customer orders in Round I, the trades each dealer initiate depends on the customer order received in Round I.
- Dealers trade among themselves and with the broker. it takes time for the public information concerning income becomes reflected in dealer quotes. Information about income is transmitted to dealers via customer orders in Round I.
- Each dealer then has some information about the aggregate income at the start of Round II but the information is imprecise. The optimal strategy is for all dealers to quote the same price and base this quote solely on the common information available in Round I. The reason is that it would otherwise expose the dealer to arbitrage trading losses. This further implies that the spot exchange rate must be unchanged between Round I and II even though dealers have some information about the aggregate income.
- Information aggregation (transmitting private imprecise information about the customer income between dealers) takes place via interdealer trades in Round II. Interdealer order flows convey information on aggregate income that becomes common information to dealers towards the end of Round II.
- At the end of Round II, the dealers and the broker observe the aggregate interdealer order flow $X_t = \sum_{d=1}^{D} T_{d,t}^{II}$.
- The retail market reopens.
- The broker and the dealers simultaneously and independently quote new prices $S_{d,t}^{III} = S_{B,t}^{III}$, different from the quotes in Round I and II.
- Investors observe these prices and place orders with dealers $T_{n,t}^{III}$.
- Dealers also trade on the interbank market $T_{d,t}^{III}$ and when each dealer has filled the customer orders, the dealer can trade with the broker. Since the dealers have learned about the aggregate interdealer order flows from Round II, this order flow is incorporated into the price quoted at the start of Round III. In particular, it is the unexpected aggregate order flow that determines the quotes. It is this transmission of information that is the distinguishing feature of the microstructure model.
- Dealers share overnight risk with customers and the broker. Customers will-

ingly absorbs all dealer inventory imbalances so that all dealers and the broker's foreign exchange holdings overnight are zero. This is consistent with actual behavior of dealers and ensures that there is no incentive for further trades in Round III.

- The order flow between customers and dealers in Round I incorporates new information relevant for the determination of the spot exchange rate. Customers are assumed to have this information and then it is transmitted to dealers through order flows.
- Dealers learn about how customers value currencies through their willingness to sell or buy currencies at quoted prices. Dealers cannot explore this information as it is not publicly known, i.e., known by other dealers.
- In the model, interdealer order flows in Round II (that are observable) conveys new information which will be used when quoting prices in Round III. Dealers do not want to hold foreign exchange overnight, overnight risk can be more efficiently shared by investors.
- The quotes in Round III must then be such that investors would like to hold the entire existing stock of foreign exchange. It would not be possible for dealers to calculate the existing stock of foreign exchange. At the same time the model implies that customer income is the only source of price changes. Thus, dealers can infer from customer income what aggregate overnight foreign exchange position that investors must be induced to hold.
- Aggregate interdealer order flows in Round II conveys information about customer income and therefore also the overnight positions investors must hold. Interdealer order flows convey information about the shift in the portfolios of investors needed to achieve efficient risk–sharing (this is the portfolio shift).

Comparison between the macro based model in question (b) and the portfolio shift model.

- The Portfolio Shifts Model incorporates key features of the FX market into a micro-based model of exchange rate determination. The equilibrium spot exchange rate literally is the price for FX quoted by foreign exchange dealers. As such, the dynamics of the spot rate are driven by changes in the dealers quote decisions.
- Public price-relevant information is immediately incorporated into the exchange rate because dealers have an incentive to change their quotes as soon as the information is known.
- There is no incentive for dealers to immediately adjust their quotes to the arrival of private price-relevant information. Rather they wait until the information becomes common knowledge via their observations on aggregate interdealer order flow before adjusting their quotes.

- The change in the spot rate is closely associated with unexpected aggregate interdealer order flow.